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Designing a Negligible Reflection Coefficient for a Uniforme Panel with Compliant Layer

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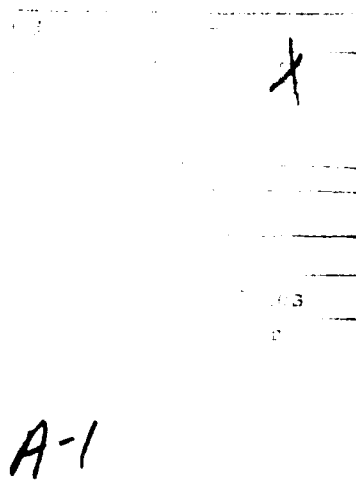**DD FORM 1473, JUN 86**   *Previous editions are obsolete.*   SECURITY CLASSIFICATION OF THIS PAGE

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ABSTRACT

The use of a *mechanical* compliant layer to achieve a *negligible* reflection coefficient atop a uniform panel facing a fluid and backed by vacuum is investigated. The material properties of the compliant layer that need to be maintained to achieve the set goal are specified in terms of the surface stiffness and the loss factor of the layer. A necessary relationship between these quantities emerges.

ADMINISTRATIVE INFORMATION

INTRODUCTION

In Part I the (specular) reflection coefficient of a plane dynamic system, immersed in fluids on both sides, is derived [1]. The plane dynamic system consists of a basic uniform panel that may be stratified by uniform passive mechanical layers. In this report, Part II, the formalism developed in Part I is applied to a simple plane dynamic system. The dynamic system selected for consideration in this paper consists merely of the basic panel topped by a compliant layer and a fluid. On the bottom side, the passive mechanical layers and the fluid are absent. The plane dynamic system and its environment are depicted in Figure 1. The purpose of this report is to explore the nature of the reflection coefficient of this simple complex. The investigation concentrates on the reduction that the compliant layer may be able to achieve in the magnitude of the reflection coefficient, and the kind of material properties of the compliant layer that the reduction calls for.

Finally, a brief examination of the radiative properties is conducted in order to exemplify the use of the formalism to ensure that the selected control of the reflective properties are not implemented at the expense of the corresponding radiative properties [1].

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GENERAL REQUIREMENTS AND BASIC EXPRESSIONS

It may be in order to initiate the specific investigations with some general ones. For this purpose Reference 1 is recalled. The (specular) reflection coefficient $R_1^0(\underline{k}_1, \omega)$ of a plane dynamic system at the interface with the top fluid is given by

$$R_1^0(\underline{k}_1, \omega) = [Z_1(\underline{k}_1, \omega) - Z_{p1}(\underline{k}_1, \omega)] \, [Z_1(\underline{k}_1, \omega) + Z_{p1}(\underline{k}_1, \omega)]^{-1} \quad , \tag{1a}$$

where $Z_1(\underline{k}_1, \omega)$ is the surface impedance of the plane dynamic system + the bottom fluid as perceived by the top fluid, and $Z_{p1}(\underline{k}_1, \omega)$ is the surface impedance of the fluid in the plane of the dynamic system [1]. The incident pressure of a plane wave is defined by $\{\underline{k}_1, k_{z1}, \omega\}$, where $\underline{k}_1$ is the "incident wavevector" in the plane of the dynamic system, and ω is the frequency variable [1]. A statement of the "wave equation" defines the incident wavevector to be supersonic; namely,

$$(\underline{k}_1)^2 + (\underline{k}_{z1})^2 = (\omega/c_1)^2 \quad ;$$
$$\underline{k}_1 = \{(\omega/c_1)\sin(\theta_1)\cos(\phi_1), (\omega/c_1)\sin(\theta_1)\sin(\phi_1)\} \quad , \tag{2}$$

where c_1 is the speed of sound in the top fluid and $\{\theta_1, \phi_1\}$ are the incidence angular parameters, see Figure 2. Since the incident wavevector is supersonic, it can be shown that the surface impedance of the top fluid, in the plane of the dynamic system, is real and of the form

$$Z_{p1}(\underline{k}_1, \omega) = [(\rho_1 c_1)/\cos(\theta_1)] \quad ; \qquad 0 \le \theta_1 < (\pi/2) \quad , \tag{3}$$

where $(\rho_1 c_1)$ is the characteristic impedance of the top fluid. It is convenient to normalize the surface impedances in terms of the characteristic or the surface impedance of the top fluid; e.g.,

$$\overline{Z}_1(\underline{k}_1, \omega) = [Z_1(\underline{k}_1, \omega)/(\rho_1 c_1)] \quad ; \qquad \overline{\overline{Z}}_1(\underline{k}_1, \omega, \theta_1) = \overline{Z}_1(\underline{k}_1, \omega)\cos(\theta_1) \quad , \tag{4a}$$

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respectively. Moreover, it is convenient to suppress the dependence of quantities on $\{\underline{k}_1, \omega\}$ when the dependence is obvious, and to drop the unit subscript from ρ_1 and c_1; e.g., it is convenient, in this vein, to express equation (4a) in the form

$$\overline{Z}_1 = [Z_1 / (\rho c)] \quad ; \qquad \overline{\overline{Z}}_1(\theta_1) = \overline{Z}_1 \cos(\theta_1) \ . \tag{4b}$$

With these normalization and abbreviations, equation (1a) may be equivalently expressed in the forms

$$R_1^0(\underline{k}_1, \omega) = R_1^0 = [\overline{Z}_1 \cos(\theta_1) - 1] \ [\overline{Z}_1 \cos(\theta_1) + 1]^{-1} \ , \tag{1b}$$

$$R_1^0 = [\overline{\overline{Z}}_1(\theta_1) - 1] \ [\overline{\overline{Z}}_1(\theta_1) + 1]^{-1} \ , \tag{1c}$$

where, again, it is observed that $\overline{Z}_1 \cos(\theta_1)$ $[=\overline{\overline{Z}}(\theta_1)]$ is the ratio of the surface impedance perceived by the top fluid in the interface with the plane dynamic system, and the surface impedance of the top fluid in that interface; these two surface impedances are evaluated at the incidence wavevector $\underline{k}_1$ and the frequency ω [1]. It is apparent that the composition of the plane dynamic system and the bottom fluid influence the nature and values of the surface impedance Z_1. Thus, in addition to the dependence of Z_1 on $\underline{k}_1$ and ω (or equivalently on $\{\theta_1, \phi_1, c, \omega\}$), it is a functional of the material properties of the various layers that may compose the plane dynamic system and the bottom fluid. In the formalism here pursued these material properties need be defined in terms of *lumped* surface impedances. These *lumped* surface impedances are expressed in terms of elemental surface masses, surface stiffnesses, and loss factors [1]. Of paramount interest is the description of the sensitivity of Z_1 $[\overline{Z}_1$ or $\overline{\overline{Z}}_1(\theta_1)]$ to variations in $\{\underline{k}_1, \omega\}$ as well as to variations in the material properties. In part, to facilitate this interest it is convenient to cast the surface impedance Z_1 in terms of its real and imaginary parts, namely,

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$$Z_1 = Z_{R1} + iZ_{I1} \quad , \tag{5a}$$

$$\overline{Z}_1 = \overline{Z}_{R1} + i\,\overline{Z}_{I1} \quad , \tag{5b}$$

$$\overline{\overline{Z}}_1(\theta_1) = \overline{\overline{Z}}_{R1}(\theta_1) + i\,\overline{\overline{Z}}_{I1}(\theta_1) \quad ; \quad \theta \le \theta_1 < (\pi/2) \quad , \tag{5c}$$

where the quantities Z_{R1}, $\overline{Z}_{I1}$, $\overline{Z}_{R1}$, etc., are all real. One may then represent $Z_1 [\overline{Z}_1 \text{ or } \overline{\overline{Z}}(\theta_1)]$ on the complex plane and graphically investigate changes in these quantities with specific variations in $\{\underline{k}_1, \omega\}$ and/or the material properties and/or the composition of the layers. If the plane dynamic system is passive, as is assumed in this paper, then $Z_{R1} [\overline{Z}_{R1} \text{ or } \overline{\overline{Z}}_{R1}(\theta_1)]$ is invariably positive; the positive value indicates that this term is resistance controlled. On the other hand, $Z_{I1} [\overline{Z}_{I1} \text{ or } \overline{\overline{Z}}_{I1}(\theta_1)]$ may assume either a positive or a negative value; the positive value indicates that this term is mass controlled and the negative value indicates that this term is stiffness controlled. A simple example of such graphical investigation is depicted in Figure 3. From equations (1) and (5) one obtains

$$R_1^0 = \left\{ [\,|\overline{\overline{Z}}_1(\theta_1)|^2 - 1] + 2i\,\overline{\overline{Z}}_{I1}(\theta_1) \right\} \left\{ [\,|\overline{\overline{Z}}_1(\theta_1)|^2 + 1] + 2\overline{\overline{Z}}_{R1}(\theta_1) \right\}^{-1} \quad . \tag{6}$$

A few asymptotic cases to illustrate the nature of equation (6) may be of interest:

1. The plane dynamic system consists of a free plane and the bottom fluid matches the top fluid. In this case

$$\overline{Z}_1(\theta_1) = 1 \quad , \tag{7a}$$

and from equation (6) one obtains then

$$R_1^0 = 0 \quad , \tag{8a}$$

as expected.

2. The plane dynamic system consists of a free plane and the bottom fluid is absent altogether. In this case

$$\overline{\overline{Z}}_1(\theta_1) = 0 \quad , \tag{7b}$$

and from equation (6) one obtains then

$$R_1^0 = -1 \quad , \tag{8b}$$

which is consistent with the reflection coefficient of "a pressure released boundary."

3. The plane dynamic system is lightly damped and the bottom fluid is either absent or the incident wavevector $\underline{k}_1$ is subsonic with respect to the speed of sound in the bottom fluid. In this case

$$\overline{\overline{Z}}_{R1}(\theta_1) \ll 1 \quad , \tag{7c}$$

and from equation (6) one obtains then

$$|R_1^0| \simeq 1 \quad , \tag{8c}$$

independent of the value of $\overline{\overline{Z}}_{I1}(\theta_1)$.

4. The plane dynamic system consists of a number (≥ 2) of layers. The surface impedances of these layers possess terms that are mass and stiffness controlled. The composition of the layers is designed such that these terms cancel out leaving the resulting surface impedance Z_1 free of an imaginary term; cancellation of this kind is commonly termed resonance. In this resonance case

$$\bar{\bar{Z}}_{I1}(\theta_1) << 1 \quad , \tag{7d}$$

and from equation (6) one obtains then

$$R_1^0 \simeq [\bar{\bar{Z}}_{R1}(\theta_1) - 1][\bar{\bar{Z}}_{R1}(\theta_1) + 1]^{-1} \quad . \tag{8d}$$

If, in addition, the damping is light and the bottom fluid is either absent or nonpropagating for the specific incidence wavevector $\underline{k}_1$, so that one can also insure that

$$\bar{\bar{Z}}_{R1}(\theta_1) << 1 \quad , \tag{7e}$$

then from equation (8d) one obtains

$$R_1^0 \simeq 1 \quad , \tag{8e}$$

which is consistent with equation (8c), as it should be.

5. The plane dynamic system is resonant and is substantially damped for the spectral component at $\{\underline{k}_1, \omega\}$ of the incident plane wave. If the resonance and the high damping renders

$$\bar{\bar{Z}}_{I1}(\theta_1) << 1 \, , \text{ and } \bar{\bar{Z}}_{R1}(\theta_1) \simeq 1 \quad , \tag{7f}$$

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then from equation (6) or simply from equation (8d) one obtains

$$R_1^0 \simeq 0 \quad .$$

(8f)

Equations (7f) and (8f) are of basic significance to this paper. This example (No. 5) reveals a specific use for the presentation of the surface impedance $Z_1[\overline{Z}_1 \text{ or } \overline{Z}(\theta_1)]$ on the complex plane. It is apparent that to achieve a negligible reflection coefficient; $R_1^0 \Rightarrow 0$, one needs compose a plane dynamic system for which the imaginary part of $\overline{\overline{Z}}_1(\theta_1)$; namely, $Z_{I1}(\theta_1)$, vanishes and the real part; namely, $\overline{\overline{Z}}_{R1}(\theta_1)$, is placed at unity. The position $\{1, 0\}$ on the complex plane is thus called bull's-eye with respect to $\overline{\overline{Z}}_1(\theta_1)$; see Figure 3a. With respect to the normalized surface impedance $\overline{Z}_1$, the bull's-eye is at the position $\{[\cos(\theta_1)]^{-1}, 0\}$, which is a function of the angle of incidence; see Figure 3b.

Supplemental to the estimation of the reflection coefficient there is interest in the sensitivity of the reflection coefficient to various changes that may influence the values of $Z_1[\overline{Z}_1 \text{ or } \overline{\overline{Z}}_1(\theta_1)]$. From equation (1) it is observed that an incremental change $\Delta\overline{\overline{Z}}_1(\theta_1)$ in the normalized surface impedance of the plane dynamic system + the bottom fluid, is accompanied by an incremental change ΔR_1^0 in the reflection coefficient. These incremental changes are related in the form

$$\Delta R_1^0 = 2[\overline{\overline{Z}}_1(\theta_1) + 1]_0^{-2} \, \Delta\overline{\overline{Z}}_1(\theta_1) \quad ,$$

(9)

where the subscript 0 to brackets (or parenthesis) indicates that the quantities are to be evaluated without the change; i.e., the original values that existed prior to the change are to be maintained. The incremental change in $\overline{\overline{Z}}_1(\theta_1)$ may now be expressed in terms of various dependencies, gross or detailed, as the case may be. In particular

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$$\Delta \, \overline{\overline{Z}}_1(\theta_1) = \cos(\theta_1)(\rho c)^{-1} \Delta Z_1 \; [\text{with } (\theta_1)_0 \text{ and } (\rho c)_0]$$

$$+ \, \overline{Z}_1 \, \Delta \cos(\theta_1) \; [\text{with } (Z_1)_0 \text{ and } (\rho c)_0]$$

$$- \, \overline{\overline{Z}}_1(\theta_1)(\rho c)^{-1} \Delta(\rho c) \; [\text{with } (\theta_1)_0 \text{ and } (Z_1)_0] \quad .$$

$$\tag{10}$$

Recalling example No. 5, inserting equation (7f) and (8f) in equations (9) and (10), and fixing the original values of (θ_1) and (ρc), one obtains

$$\Delta R_1^0 \simeq (1/2) \cos(\theta_1) \Delta \overline{Z}_1 \, [(\theta_1) \text{ and } (\rho c)_0] \quad ; \qquad \Delta R_1^0 \simeq 0 \quad .$$

$$\tag{11}$$

Under these conditions and definitions, the incremental change $\Delta \overline{Z}_1$ is less than halved into an incremental change in the reflection coefficient. As just discussed, when detailed dependences of $\overline{\overline{Z}}_1(\theta_1)$, on quantities and parameters that define the plane dynamic system and the fluids, become more explicitly available, the information can be infused not only into the estimation of the reflection coefficient itself, but also into the determination of its sensitivity to modifications in these quantities and parameters.

Attention is now turned to the analysis of a specific and a simple situation. This section was advanced to emphasize that the analysis to be presented subsequently is merely a specific and a simple example of a more general one. One recognizes that a relationship between a general analysis and a specific one is usually of help to both.

REFLECTION COEFFICIENT OF A UNIFORM PANEL

Interest in this paper focuses on the investigation of the reflection coefficient of a plane dynamic system composed of a basic panel that is stratified atop by a compliant layer. The plane dynamic system is facing a fluid atop and vacuum on the bottom side; see Figure 1. It may be useful to investigate first the reflection coefficient of a plane dynamic system that is merely composed of a basic (uniform) panel. The analytical model of a basic panel with fluid atop and

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vacuum on the bottom side is sketched in Figure 4. In Figure 5 the equivalent circuit diagram for this model is presented. The surface impedance of the panel is designated $Z_p(\underline{k}, \omega)$ and the fluid is defined by a density ρ and a speed of sound c. The normalized surface impedance $\overline{Z}_1(\underline{k}_1, \omega)$, as perceived by the fluid into the interface with the plane dynamic system, is simply

$$\overline{Z}_1 = \overline{Z}_p \quad ; \qquad \overline{Z}_p = [Z_p(\underline{k}_1, \omega)/(\rho c)] \quad ; \qquad \overline{\overline{Z}}_1(\theta_1) = \overline{Z}_p \cos(\theta_1) \ . \tag{12}$$

[cf. equation (4).] To determine the reflection coefficient of this uniform panel, it is necessary to define the surface impedance $Z_p(\underline{k}_1, \omega)$ in an explicit form. In Part I it was argued that the *lumped* surface impedance form is the one that needs to be stated. The lumped surface impedance of a uniform panel may be stated in the form

$$\overline{Z}_p(\underline{k}, \omega) \simeq i(\omega/\omega_c)(\epsilon_c)^{-1}[1 - (|\underline{k}|/k_p)^{2P}] \quad ; \qquad \epsilon_c = (\rho c/\omega_c m) \quad ;$$

$$k_p = k_{p0}(1 - i\eta_p) \ , \tag{13}$$

where

$$k_{p0}^2 = \begin{cases} (\omega/c_\ell)^2 \quad ; \qquad P = 1 \quad , \qquad \text{(a membrane in flexure)} \ , & \text{(14a)} \\[3mm] (\omega\omega_c/c^2) \quad ; \qquad P = 2 \quad , \qquad \text{(a plate in flexure)} & \text{(14b)} \end{cases}$$

m is the mass per unit area, ϵ_c is the fluid loading parameter, η_p is the stiffness controlled loss factor, k_{p0} is the free wavenumber of the panel, the panel is assumed isotropic, and ω_c is a normalizing frequency if equation (14a) is valid, and is the critical frequency if equation (14b) is valid [1]. The critical frequency ω_c is defined with respect to the speed of sound c in the fluid. [For the longitudinal response in a plate, equation (14a) is valid, however, a Poisson's ratio needs

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to be appropriately inserted. Such considerations are, however, beyond the scope of this report.]
From equations (3), (4), (12), and (13) one obtains

$$\overline{\overline{Z}}_1(\theta_1) = i(\omega/\omega_c) \left[\cos(\theta_1)/\varepsilon_c\right] \left\{1 - \left[(\omega/ck_{p0}) \sin(\theta_1)(1 - i\eta_p)^{-1}\right]^{2P}\right\} \quad . \tag{15}$$

For most normal situations of interest

$$(\omega/ck_{p0}) \sin(\theta_1) \ll 1 \quad \text{and} \quad \eta_p \ll 1 \quad , \tag{16}$$

so that equation (15) can be reasonably approximated to read

$$\overline{\overline{Z}}_1(\theta_1) \simeq i\overline{\overline{Z}}_{I1}(\theta_1) \simeq i(\omega/\omega_c) \left[\cos(\theta_1)/\varepsilon_c\right] \quad ; \qquad \overline{\overline{Z}}_{R1}(\theta_1) \ll 1 \quad . \tag{17}$$

Thus the normalized surface impedance $\overline{Z}_1$ is substantially mass controlled and the damping term is small. From equations (1c), (6), and (17) the reflection coefficient for the model depicted in Figures 4 and 5 is found to be

$$R_1^0 \simeq \left[\overline{\overline{Z}}_{I1}(\theta_1) - i\right] \left[\overline{\overline{Z}}_{I1}(\theta_1) + i\right]^{-1} \quad ;$$

$$|R_1^0| \simeq 1 \quad . \tag{18}$$

[cf. example No. 3.] The task of the compliant layer is now clearer. Can the introduction of the compliant layer cause the absolute value of the reflection coefficient to become negligible compared with unity? In view of equation (18), one must admit that such an achievement would be remarkable even if this negligible value were maintained only over a limited range of frequency and angle of incidence.

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INTRODUCTION OF A COMPLIANT LAYER

The introduction of a compliant layer is depicted in Figure 1. [cf. Figure 4.] The surface impedance of the compliant layer is designated $Z_c(\underline{k}, \omega)$. The equivalent circuit diagram for the model depicted in Figure 1 is presented in Figure 6. The normalized surface impedance $\overline{Z}_1(\underline{k}_1, \omega)$, as perceived by the fluid into the interface with the plane dynamic system, is given by

$$\overline{Z}_1 = [Z_1/(\rho_1 c_1)] = \overline{Z}_p \overline{Z}_c [\overline{Z}_p + \overline{Z}_c]^{-1} \quad , \tag{19}$$

where

$$\overline{Z}_c = [\overline{Z}_c(\underline{k}_1, \omega)/(\rho c)] \quad , \tag{20}$$

and $\overline{Z}_p$ is as stated in equation (12). The explicit form of the *lumped* surface impedance of the basic panel is furnished in equation (13). The explicit form of the *lumped* surface impedance of the compliant layer may be stated in the normalized form

$$\overline{Z}_c(\underline{k}, \omega) = - i (\omega_0^2/\omega\omega_c) (\varepsilon_c)^{-1} (1 + i\eta_c) \quad ; \qquad \omega_0^2 = (K/m) \quad , \tag{21}$$

where ω_0 is the resonance frequency of a resonant dynamic system consisting of the surface mass m of the panel and the surface stiffness K of the compliant layer, and η_c is the loss factor in the compliant layer [1]. In terms of the analysis adopted in this paper, K and η_c fully define the material properties of the compliant layer; any further details in the material properties of the compliant layer are not needed for the present analysis. From equations (14b), (15), (19), and (21) one obtains

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$$\overline{Z}_1 \simeq -\, i(\omega_0^2/\omega\omega_c\varepsilon_c)\,(1 + i\eta_c)\,[1 - (\omega/\omega_c)^2\sin^4(\theta)\,(1 + i\eta_d)]$$

$$\left[1 - (\omega_0/\omega)^2 - (\omega/\omega_c)^2\sin^4(\theta) + i(\omega_0/\omega)^2\left\{\eta_c + (\omega^2/\omega_0\omega_c)\sin^4(\theta)\,\eta_d\right\}\right]$$

$$\left\{[1 - (\omega_0/\omega)^2 - (\omega/\omega_c)^2\sin^4(\theta)]^2 + (\omega_0/\omega)^4[\eta_c + (\omega/\omega_0\omega_c)^2\sin^4(\theta)\,\eta_d]\right\}^{-1} \quad , \qquad (22)$$

where $\eta_d \simeq 4\eta_p$, and it is assumed that $\eta_d \ll 1$. The choice of inserting equation (14b) in equation (19), rather than equation (14a), is merely a matter of convenience and use. The expression in equation (22) is quite cumbersome. It is possible to reduce this cumbersomeness if one may assume that

$$(\omega/\omega_c)^2\,\sin^4(\theta_1) \ll 1 \quad . \qquad (23)$$

[cf. equation (16).] When equation (23) is imposed on equation (22) one obtains

$$\overline{Z}_1 \simeq -\, i(\omega_0^2/\omega\omega_c\,\varepsilon_c)\,[1 - (\omega_0/\omega)^2(1 + \eta_c^2) + i\eta_c]$$

$$\left\{[1 - (\omega_0/\omega)^2]^2 + (\omega_0/\omega)^4\,\eta_c^2\right\}^{-1} \quad . \qquad (24)$$

The condition stated in equation (23) is subsequently assumed and, therefore, equation (24) subsequently stands. This assumption does not permit the formalism to be carried out at high frequency $(\omega/\omega_c) \gtrsim 5 \times 10^{-1}$ and/or at grazing angles of incidence $\theta_1 \simeq (\pi/2)$. [It is recognized, however, that when either of these conditions is violated, the simple formalism employed herein need be revamped in any case.] Now that the bull is had by the horns through the derivation of equation (24), one may inquire whether this equation can yield the bull's-eye; i.e., can equation (7f) be satisfied by equation (24)? Imposing equation (7f) on equation (24) yields

$$(\omega/\omega_0)^2 = (1 + \eta_c^2), \quad \text{or equivalently,} \quad (\omega/\omega_c)^2 = (\omega_0/\omega_c)^2(1 + \eta_c^2) \quad , \qquad (25a)$$

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$$(\omega/\omega_c) \, (\varepsilon_c \eta_c)^{-1} \cos(\theta_1) = 1, \qquad \text{or equivalently,} \qquad (\omega m/\rho c) \cos(\theta_1) = \eta_c \quad , \tag{25b}$$

and it is emphasized that to achieve a vanishing reflection coefficient as stated in equation (8f), the two relationships; equations (25a) and (25b), need to be satisfied simultaneously. When these relationships are satisfied, a bull's-eye is achieved in the displays that are presented in the format of Figure 3. Figures 7 through 9 are testimonials to such achievements. Equation (25a) relates the resonance frequency to the resonance frequency ω_0 in the absence of damping, and the damping loss factor η_c. This relationship is well known in structural dynamics and, therefore confirms that the phenomenon involved in achieving a vanishing reflection coefficient is indeed a resonance based phenomenon. On the other hand, equation (25b) is novel, if not strange , to say the least. This equation states that the ratio of the absolute value of the surface (mass) impedance of the basic panel to the surface impedance of the fluid at incidence is equal to the loss factor of the compliant layer.

A legitimate question may be posed at this stage. Is there a physical compliant layer that possesses reasonable material properties that will satisfy equation (25)? Figures 7 through 9 indicate, to a degree, that the question can be answered affirmatively; nonetheless, a more thorough answer is demanded. Before answering this important question, it may be in order to introduce a compliant layer on the panel and investigate its influence on the radiated pressure generated by an external drive acting on the basic panel; see Figures 5 and 6.

INFLUENCE OF A COMPLIANT LAYER ON THE RADIATED PRESSURE

From Reference 1 and/or Figures 5 and 6, the ratio $R_{1c}(\underline{k}_1, \omega)$ of the radiated pressures in the far field, with and without the compliant layer, respectively, is found to be

$$R_{1c}(\underline{k}, \omega) = \bar{Z}_1(\underline{k}, \omega) \, [1 + \bar{Z}_1(\underline{k}, \omega) \cos(\theta)]^{-1} \, \left\{ \bar{Z}_p(\underline{k}, \omega) \, [1 + \bar{Z}_p(\underline{k}, \omega) \cos(\theta)]^{-1} \right\}^{-1} ;$$

$$0 \le \theta < (\pi/2) \quad , \tag{26a}$$

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or equivalently

$$R_{1c}(\underline{k}, \omega) = \overline{\overline{Z}}_1(\underline{k}, \omega, \theta) \; [\overline{\overline{Z}}_1(\underline{k}, \omega, \theta) + 1]^{-1} \; \left\{ \overline{\overline{Z}}_p(\underline{k}, \omega, \theta) \; [\overline{\overline{Z}}_p(\underline{k}, \omega, \theta) + 1]^{-1} \right\}^{-1} , \quad (26b)$$

where the radiated pressures are generated by an external drive $P_e(\underline{k}, \omega)$ acting on the basic panel, and it is noted that

$$\overline{\overline{Z}}(\underline{k}, \omega, \theta) = \overline{Z}(\underline{k}, \omega) \cos(\theta) \quad ; \qquad [1 + (\overline{Z}_p/\overline{Z}_c)] = (\overline{Z}_1/\overline{Z}_p) \quad . \tag{27}$$

[cf. equations (4) and (5).] Equation (26) may be utilized to examine the influence on the radiative properties as the reflection coefficient is modified through the addition of a compliant layer onto the basic panel. It may then be of particular significance to assess equation (26) with the imposition of equation (7f). With this imposition, equation (26) yields

$$R_{1c}(\underline{k}_1, \omega) \Rightarrow R_{1c}^0 = [\overline{\overline{Z}}_p(\theta_1) + 1] \, [2\overline{\overline{Z}}_p(\theta_1)]^{-1} \quad ; \qquad R_1^0 = 0 \quad . \tag{28a}$$

Using equation (25b) in equation (28a) one obtains the explicit expression for R_{1c}^0 in the form

$$R_{1c}^0 \simeq (1/2\,\eta_c)(\eta_c - i) \quad ; \qquad |R_{1c}^0| = (1/2\,\eta_c)(1 + \eta_c^2)^{1/2} \quad . \tag{28b}$$

It is noted that if $|R_{1c}^0|$ exceeds unity, the radiated pressure is increased by the introduction of the compliant layer. This increase may tend to replace the absence of the reflected pressure that is designed into the stratified compliant layer. To ensure that this radiative condition does not occur, one must impose that

$$2\eta_c \geq (1 + \eta_c^2)^{1/2} \quad , \tag{29}$$

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and that any external drive component $P_e(\underline{k}, \omega)$ that may be acting on the basic panel is not excessive. Again, equation (29) is novel and a most interesting statement; η_c must be maintained high, $\eta_c \geq (1/2)$, to ensure that the radiated pressure component $P_{rad}(\underline{k}_1, \omega)$ is not amplified by the introduction of the compliant layer; see Figures 5 and 6.

MATERIAL PROPERTIES OF THE COMPLIANT LAYER

In this section an attempt is made to define the material properties of the compliant layer that are necessary to achieve a vanishing reflection coefficient. For this purpose the frequency variable is eliminated in equation (25). The result of this elimination is

$$[(K/\omega_c)/(\rho c)] = [\varepsilon_c/\cos^2(\theta_1)] \, [\eta_c^2(1 + \eta_c^2)^{-1}] \quad . \tag{30a}$$

where, it is recalled, $[\varepsilon_c/\cos^2(\theta_1)]$ is defined in terms of parameters that belong solely to the basic panel, the fluid, and the angle of incidence; this factor is entirely independent of parameters that belong to the compliant layer. Fixing a value for $[\varepsilon_c/\cos^2(\theta_1)]$ yields a relationship between the normalized surface stiffness $[(K/\omega_c)/(\rho c)]$ and the loss factor η_c of the compliant layer. This relationship is *a necessary condition* to achieve a vanishing reflection coefficient. Examples of such relationships are depicted in Figure 10a. Another manner of stating the material properties of the compliant layer is afforded by stating equation (30a) in the alternate form

$$(\omega_0/\omega_c) \simeq [\varepsilon_c/\cos(\theta_1)] \, [\eta_c(1 + \eta_c^2)^{-1/2}] \quad ; \qquad (K/m\omega_c^2) = (\omega_0/\omega_c)^2 \quad . \tag{30b}$$

For a specific value of $[\varepsilon_c/\cos(\theta_1)]$, *a necessary (condition) relationship* can be established between the normalized resonance frequency (ω_0/ω_c) and the loss factor η_c of the compliant layer that is designated to achieve a vanishing reflection coefficient. Examples of such necessary relationships are depicted graphically in Figure 10b. Again, it is emphasized that the formalism is simplistic and, therefore, the conclusions that the formalism suggests need be taken with

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caution [1]. This is particularly relevant to situations that represent extreme values of the parameters that are involved. Reasonable parametric values need be limited to:

$\theta_1 < (\pi/2)$, $\eta_c \leq 2$, $(\omega_0/\omega_c) \geq 5 \times 10^{-2}$, and $\varepsilon_c \leq 0.2$. Combined with the criteria stated in equations (16) and (23), it is also necessary to impose that $5 \times 10^{-1} > (\omega/\omega_c) \geq 5 \times 10^{-2}$. The examples cited in Figures 7 through 10 are largely so restricted. Indeed, in the computations that generate these and the subsequent figures, the (normalized) parameters that defined the basic panel and the fluid are assigned the standard values: $\varepsilon_c = 0.10$ and $(\rho/\rho_c) = 0.13$ where ρ_p is the specific density of the basic panel

RANGES IN THE MATERIAL PROPERTIES OF THE COMPLIANT LAYER

Figure 10 depicts the loci of the material properties of the compliant layer that are necessary to yield a zero reflection coefficient; i.e., $R_1^0(\underline{k}_1, \omega) = 0$. Once $[\varepsilon_c/\cos^2(\theta_1)]$ or correspondingly $[\varepsilon_c/\cos(\theta_1)]$ is fixed, each locus in Figure 10 demands a specific value of (ω/ω_c) if indeed a vanishing reflection coefficient is to be achieved. The positions of a few such locus points are indicated in Figure 10; this figure confirms the feasibility of the declared purpose for adding a compliant layer onto the basic panel. It is recalled that the declared purpose is to achieve a negligible reflection coefficient. How *flexible* are the necessary conditions that are defined in Figure 10. Are there ranges of variations in the values of the quantities and parameters that define the reflection coefficient for which the absolute value of R_1^0 remains negligible, but not necessarily equal to zero? Some aspects of the answers to such questions were already touched upon. It was found in equation (9), for example, that the incremental changes in the normalized surface impedance $\overline{\overline{Z}}_1(\theta_1)$ cause corresponding, but subdued, changes in the reflection coefficient. A visualization of this sensitivity in the reflection coefficient R_1^0 is illustrated in Figures 7 through 9. In these figures the changes in $\overline{Z}_1$ pertaining to equal incremental changes in the three quantities and parameters that define this surface impedance are illustrated. The density of dots is grossly related to the sensitivity of the reflection coefficient to the particular parameter that is being varied, with the other two parameters held at specific values. The density of dots at and in the vicinity of

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the bull's-eye indicates the sensitivity of the reflection coefficient, to variations in these parameters, when the reflection coefficient is negligible. More precisely, equations (9) and (11) indicate that if one is satisfied with a reflection coefficient R_1^0 such that $|R_1^0| \leq 0.1$, say, then tolerated variations in $|\bar{Z}_1|$ may exceed two (2) times (0.1). Now that the parametric dependences of $R_1^0(\underline{k}_1, \omega)$ on the quantities and parameters that define the basic panel, the compliant layer, the fluid, and the incidence are stated explicitly, one may attempt, by computational illustrations, to answer questions relating to ranges in the frequency and material properties of the compliant layer. Illustrations of this kind will range the frequency and the material properties of the compliant layer that can be tolerated without violating the goal set forth for the reflection coefficient; namely, $|R_1^0| \leq 0.1$. Such illustrations are mundane but nonetheless offered in Figures 11 and 12. In Figure 11 the ranges of the material properties pertaining to three chosen values of the normalized frequency (ω/ω_c) are shown. Figure 11a is presented in the format of Figure 10a and Figure 11b in the format of Figure 10b. It is apparent from Figure 11 that a range exists. This range is limited to a region on the $\{[(K/\omega_c)/(\rho c), \eta_c]\}$ – plane in Figure 11a, and, equivalently, on the $\{(\omega_0/\omega_c), \eta_c\}$ – plane in Figure 11b. A region in each figure is dependent on the chosen value of the normalized frequency. One observes from Figure 11 that the region, on a linear scale, is larger when the normalized frequency is higher. It is further observed that the minimal values of the variables that define a region are higher when the normalized frequency is higher. Figure 11 is predicated on the validity of equation (21) and not on the manner by which this relationship is established. This relationship; i.e., equation (21), may be indigence to the structure and architecture of the compliant layer or it may be actuated by an external agent . The sole requirement is that equation (21) represents the material properties necessary to describe the *lumped* surface impedance of the compliant layer. Changes with temperature, electromagnetic settings, etc. can be accounted for by following their influences on equation (21) as such and no more [2]. In Figure 12 the range is illustrated on the $\{(\omega_0/\omega_c), (\omega/\omega_c)\}$ – plane for two chosen values of the loss factor η_c. A region (or two) on this plane is found for each value of η_c.

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It emerges from this analysis and from figures such as 11 and 12 that if one were able to specify the goal, the parameters that define the basic panel and the fluid, and the desired ranges of incidence and frequency, the formalism and the procedures here presented may, again, with caution, serve to initiate feasible designs and specifications of a compliant layer that will achieve a negligible reflection coefficient when compounded with a basic panel.

Finally, one may inquire as to whether additional stratification of the basic panel, by panel-like and compliant-like layers, can be beneficially instituted. For example, the aim of such additional stratification may be to extend the region (or regions) in which the intended goals may be attained with respect to the reflection coefficient. Additional stratification of this kind, however, are reserved for compendium parts to Parts I and II.

(ρ_1, c_1)

Z_{p1}

$2P_{I1}$

Z_c

Z_p

Fluid

Compliant layer
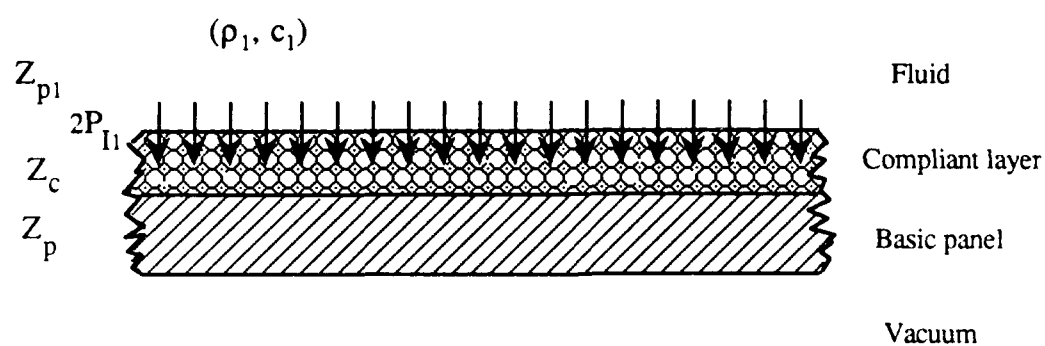
Basic panel

Vacuum

Fig. 1. A plane dynamic system consisting of a basic panel and a compliant layer facing a fluid and backed by a vacuum. Indicated are the surface impedances of the fluid, the compliant layer, and the basic panel. Also indicated is the blocked incident pressure.
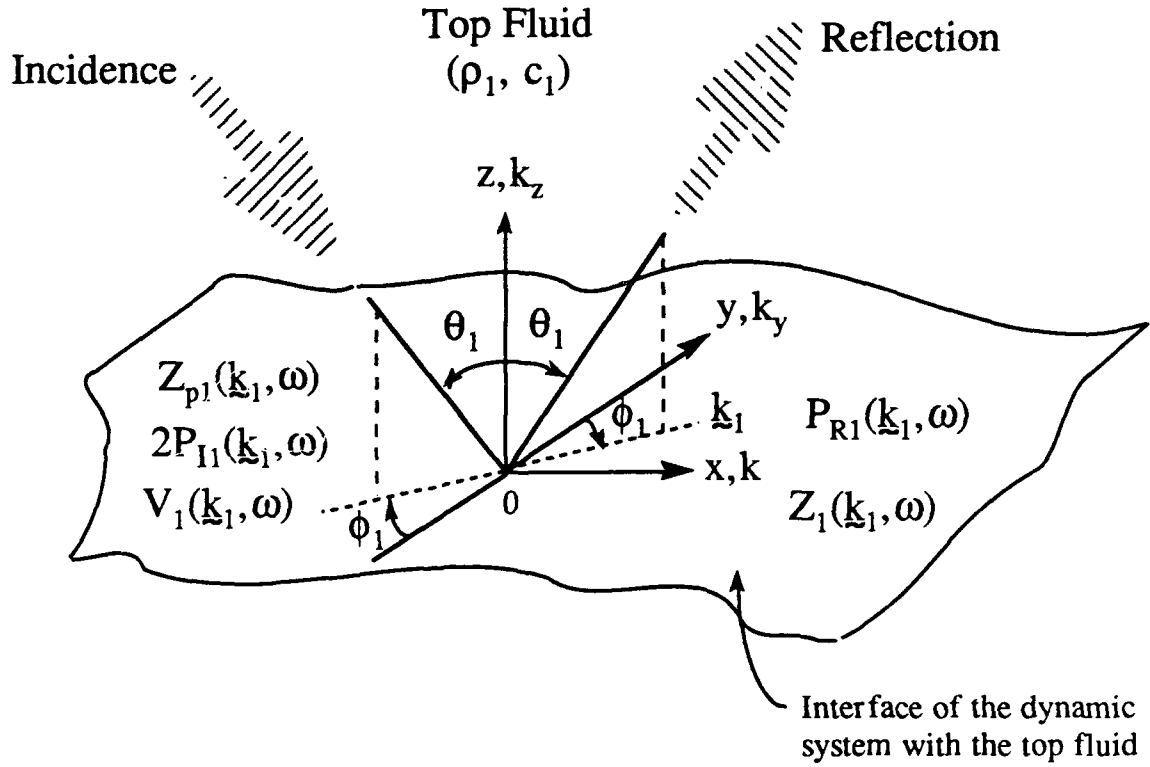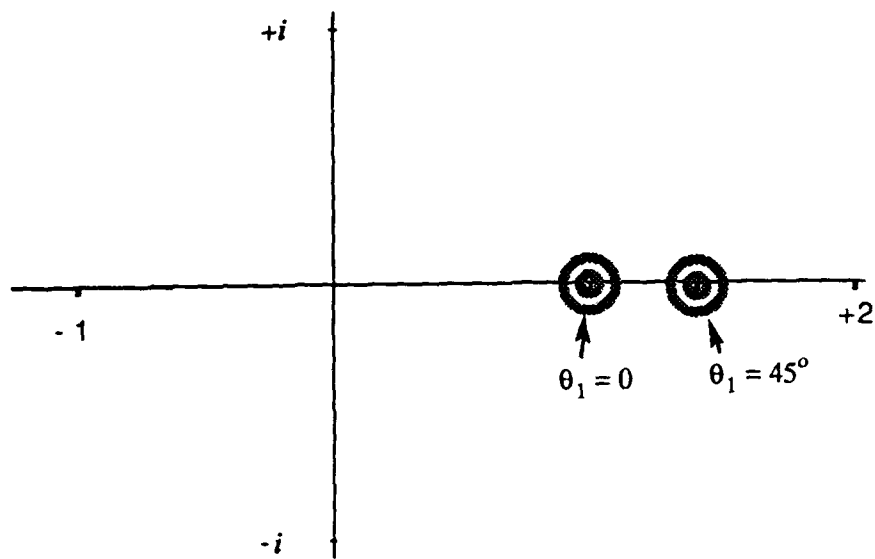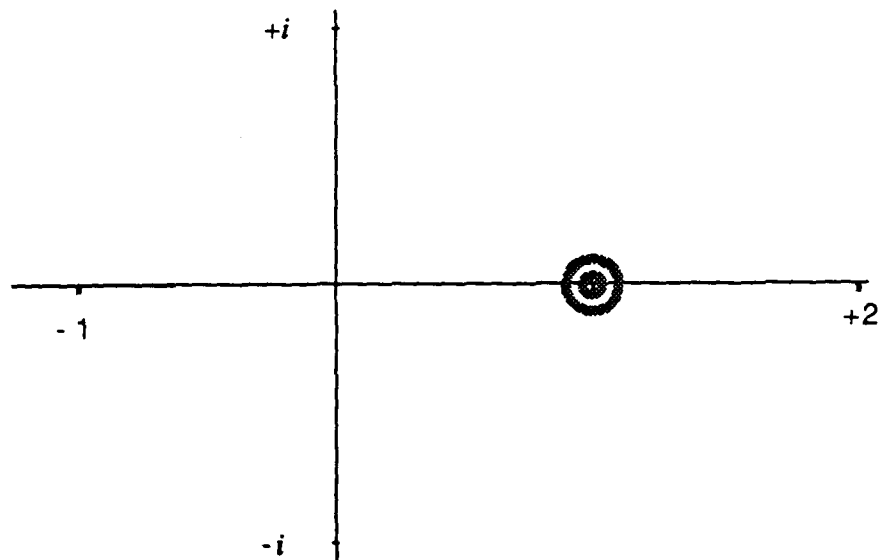
Fig. 2. Incident pressure $P_{I1}(\underline{k}_1,\omega)$, specularly reflected pressure $P_{R1}(\underline{k}_1,\omega)$, and velocity $V_1(\underline{k}_1,\omega)$ on the interface between the top fluid and the plane dynamic system. The surface impedance $Z_{p1}(\underline{k}_1,\omega)$ is that of the top fluid and $Z_1(\underline{k}_1,\omega)$ is the surface impedance perceived by the fluid in the interface.

Fig. 3. The complex plane and "bull's-eye":

    a. For depicting the normalized surface impedance $\overline{Z}_1$.

    b. For depicting the normalized surface impedance $\overline{\overline{Z}}_1(\theta_1) = \overline{Z}_1\cos(\theta_1)$.
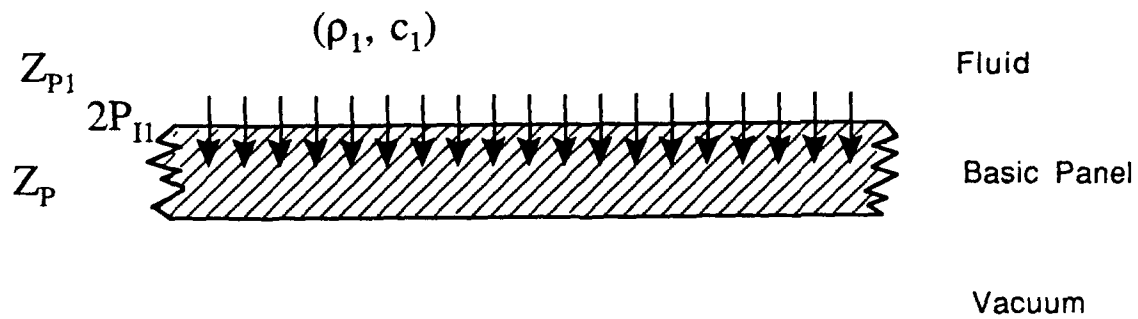
Fig. 4. A plane dynamic system consisting of a basic panel facing a fluid and backed by vacuum. The surface impedances of the fluid and the basic panel and the blocked incident pressure are indicated [cf. Fig. 1.].
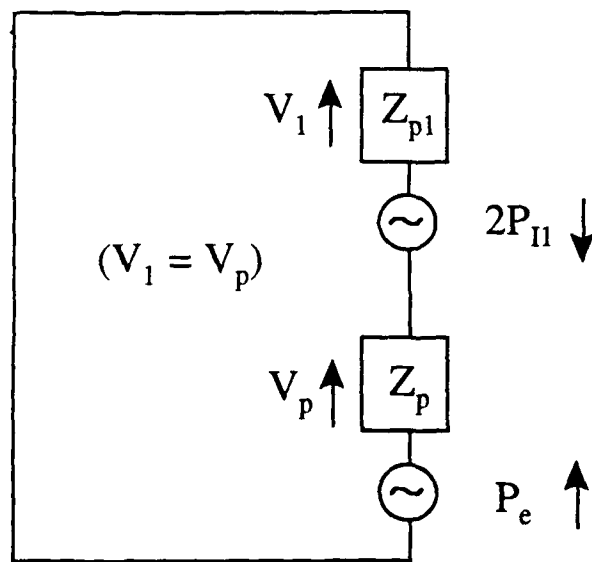

Fig. 5. Equivalent circuit diagram of the model depicted in Fig.4 showing also the external drive P_e. When $P_{I1} \neq 0$, the radiated pressure $P_{rad} = Z_{P1} V_1$.

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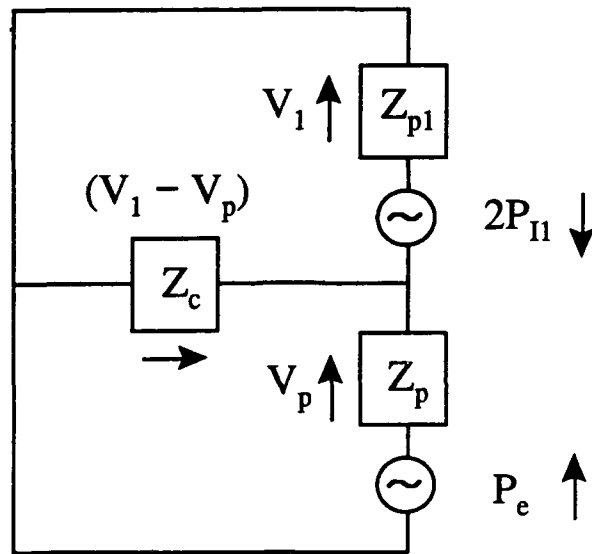
Fig. 6. Equivalent circuit diagram of the model depicted in Fig. 1 showing also the external drive P_e. When $P_{I1} \equiv 0$, the radiated pressure $P_{rad} = Z_{P1} V_1$.
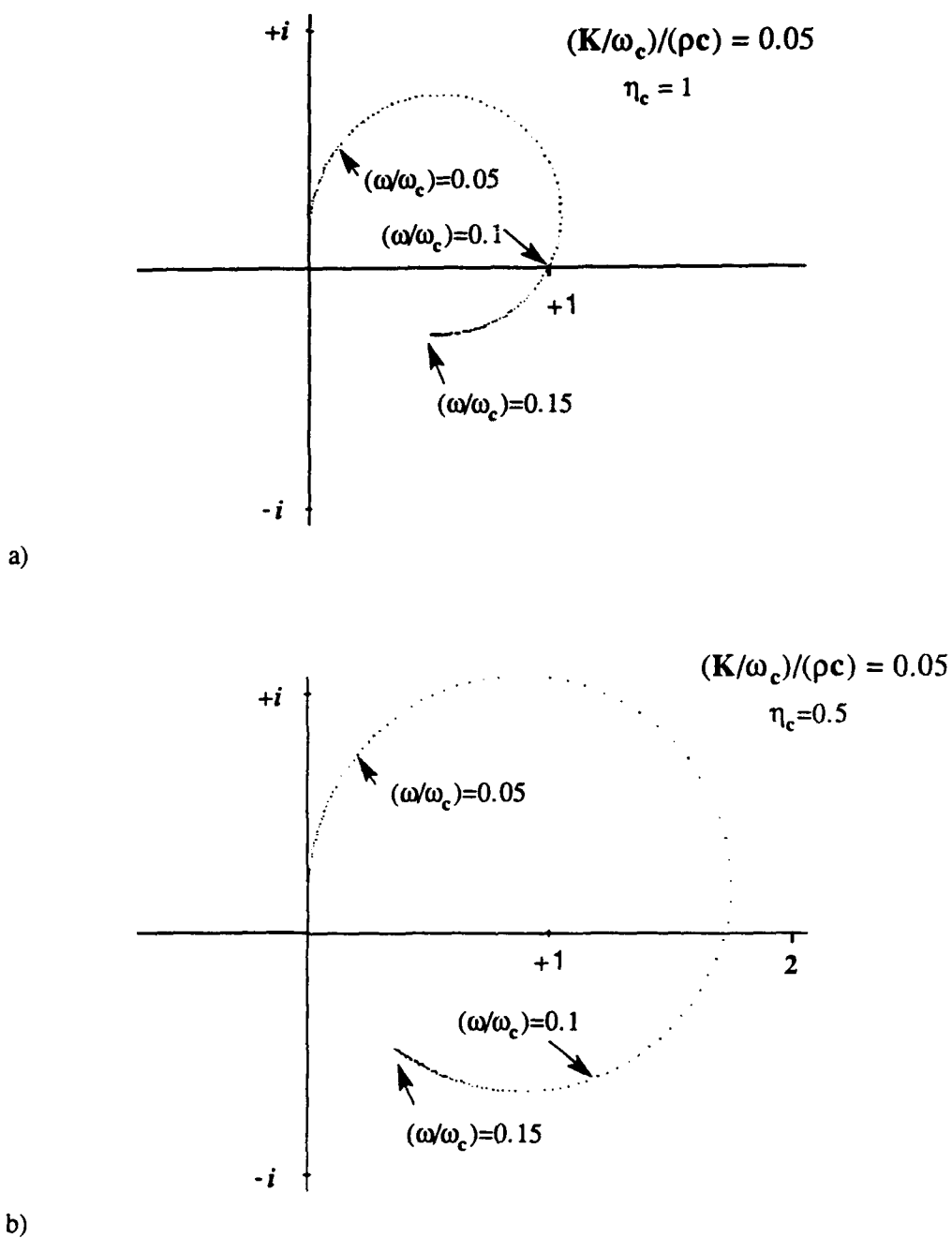
a)

b)

Fig. 7. A plot on the complex plane of the normalized surface impedance $\overline{Z}_1$ for equal
incremental step changes in the normalized frequency (ω/ω_c).

a. Bull's-eye for $(\omega/\omega_c) \cong 0.1$ if $\cos(\theta_1) \cong 1$.

b. The same as a. except that η_c is changed from 1.0 to 0.5; here a bull's-eye
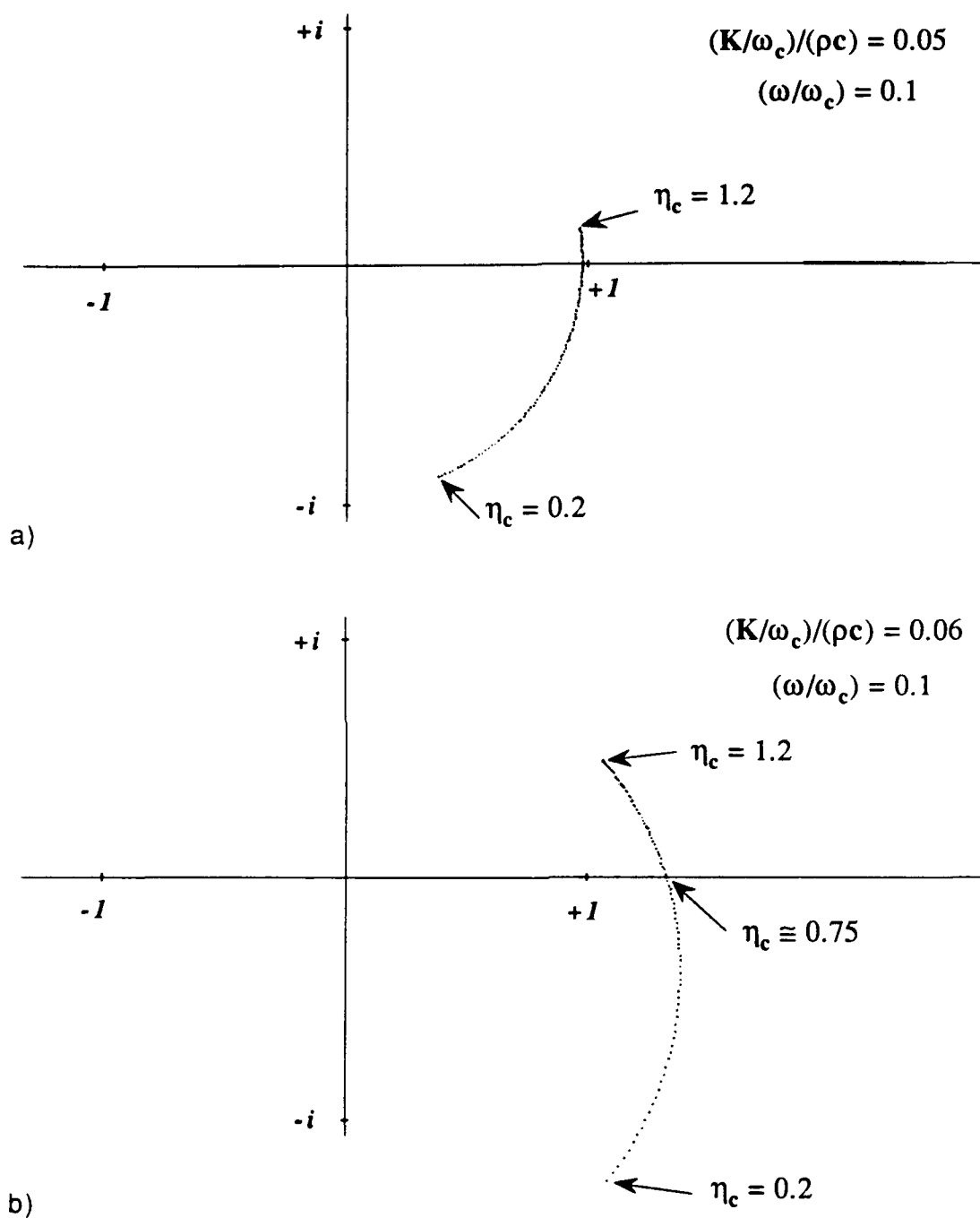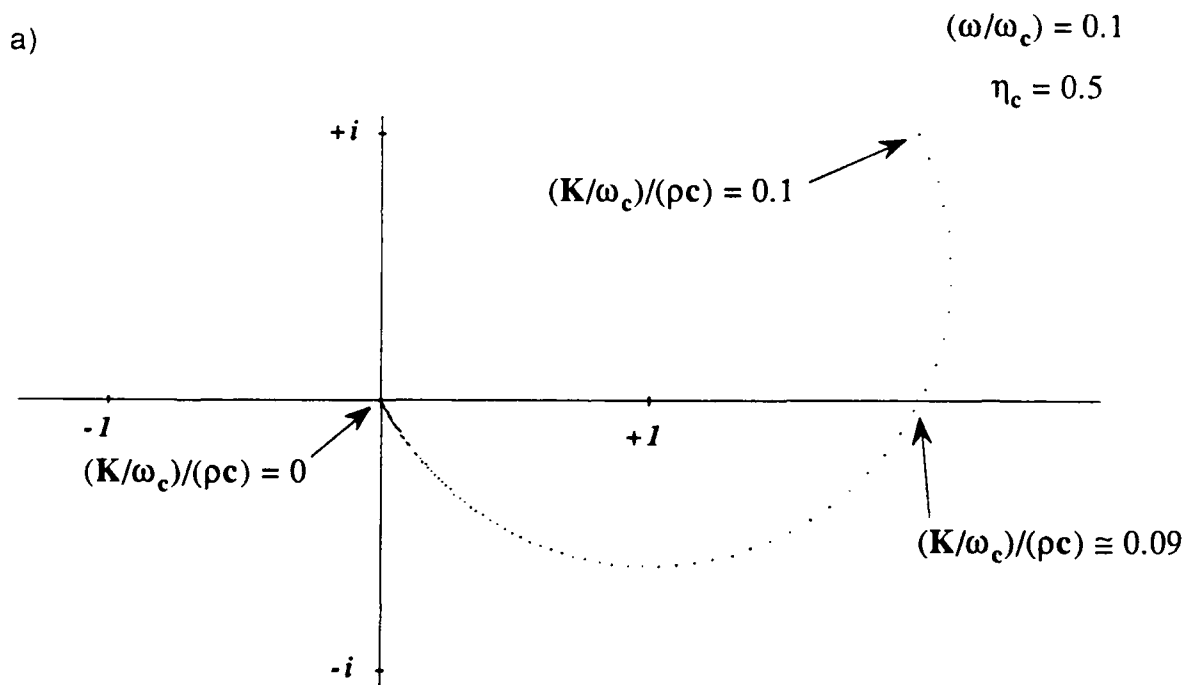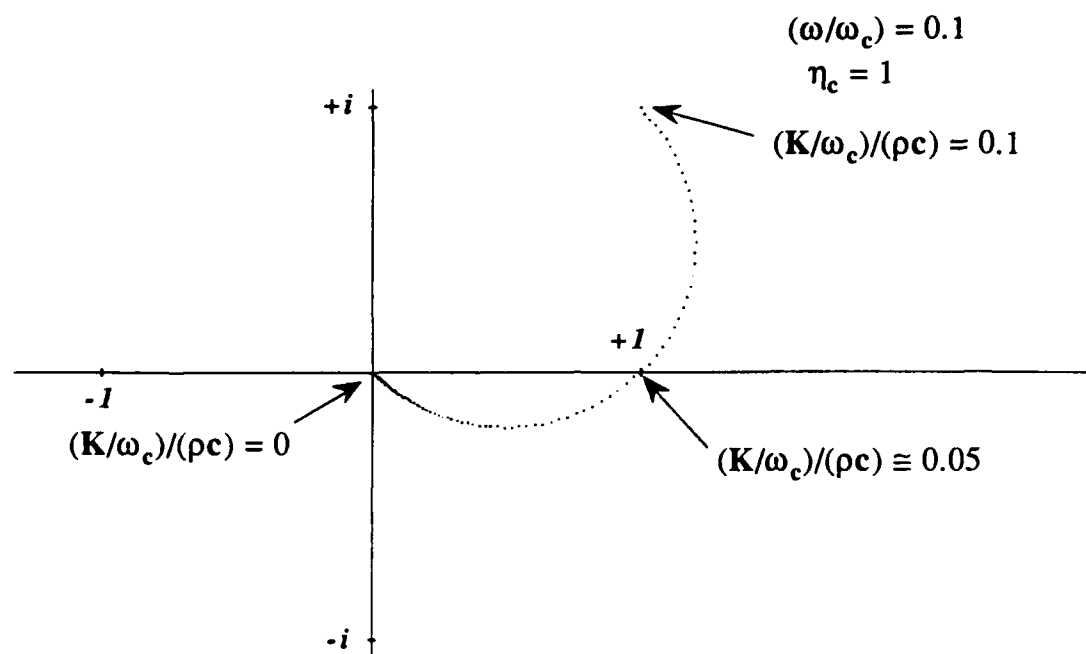occurs for $(\omega/\omega_c) \cong 0.078$ if $\cos(\theta_1) \cong 0.64$.

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Fig. 8. A plot on the complex plane of the normalized surface impedance $\overline{Z}_1$ for equal incremental step changes in the loss factor η_c.

    a. Bull's-eye for $\eta_c \cong 0.1$ if $\cos(\theta_1) \cong 1$.

    b. The same as a. except that $(K/\omega_c)/(\rho c)$ is changed from 0.05 to 0.06; here a bull's-eye occurs for $\eta_c \cong 0.75$ if $\cos(\theta_1) \cong 0.76$.

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a)

b)

Fig. 9. A plot on the complex plane of the normalized surface impedance $\overline{Z}_1$ for equal
incremental step changes in the normalized surface stiffness $(K/\omega_c)/(\rho c)$.
a. Bull's-eye for $(K/\omega_c)/(\rho c) \cong 0.05$ if $\cos(\theta_1) \cong 1$.
b. The same as a. except that η_c is changed from 1.0 to 0.5; here a bull's-eye
occurs for $(K/\omega_c)/(\rho c) \cong 0.09$ if $\cos(\theta_1) \cong 0.49$.
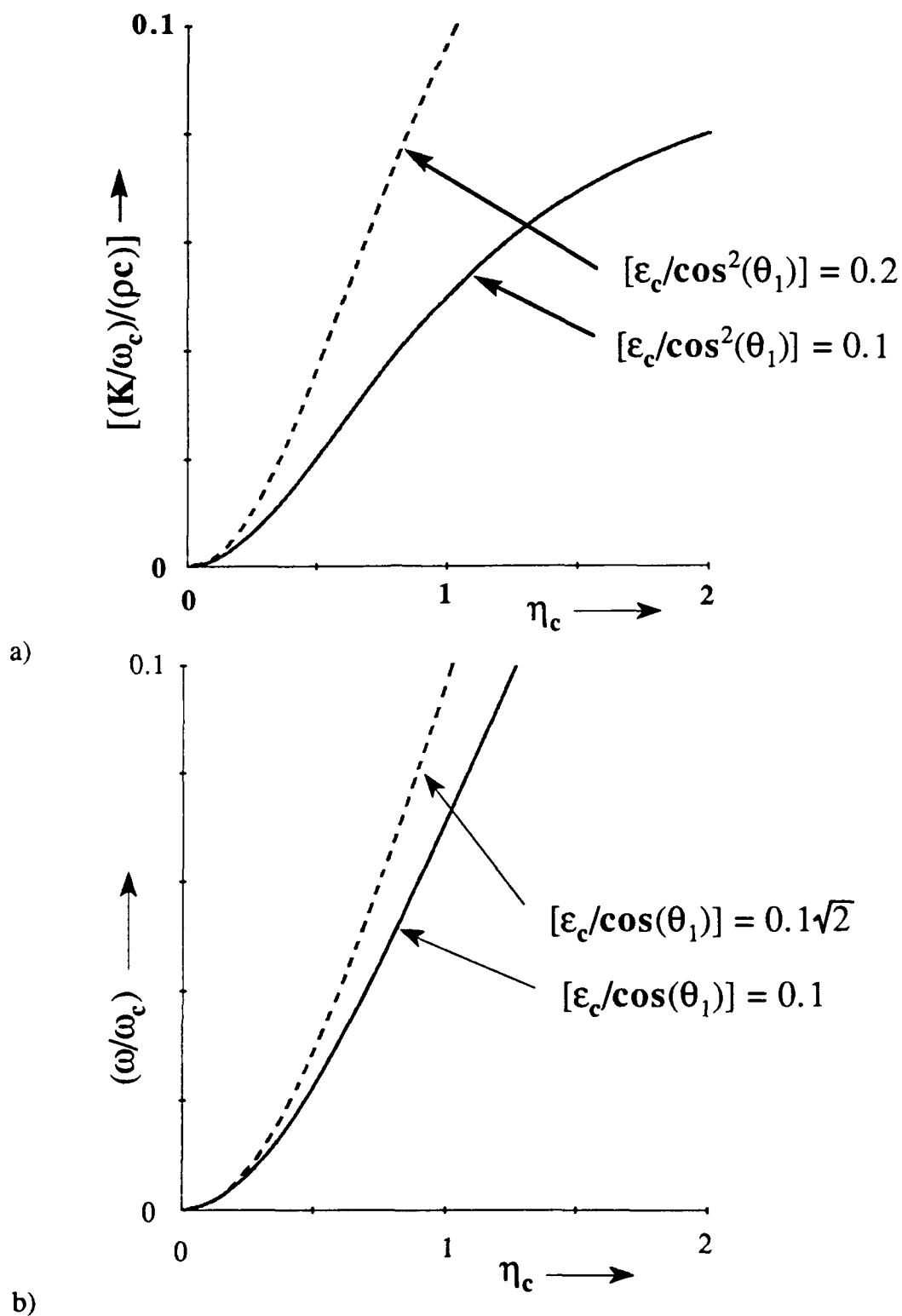
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Fig. 10 The necessary relationship defining the loci of vanishing reflection coefficient.
   a. On the $\{[(K/\omega_c)/(\rho c)], \eta_c\}$ - plane.
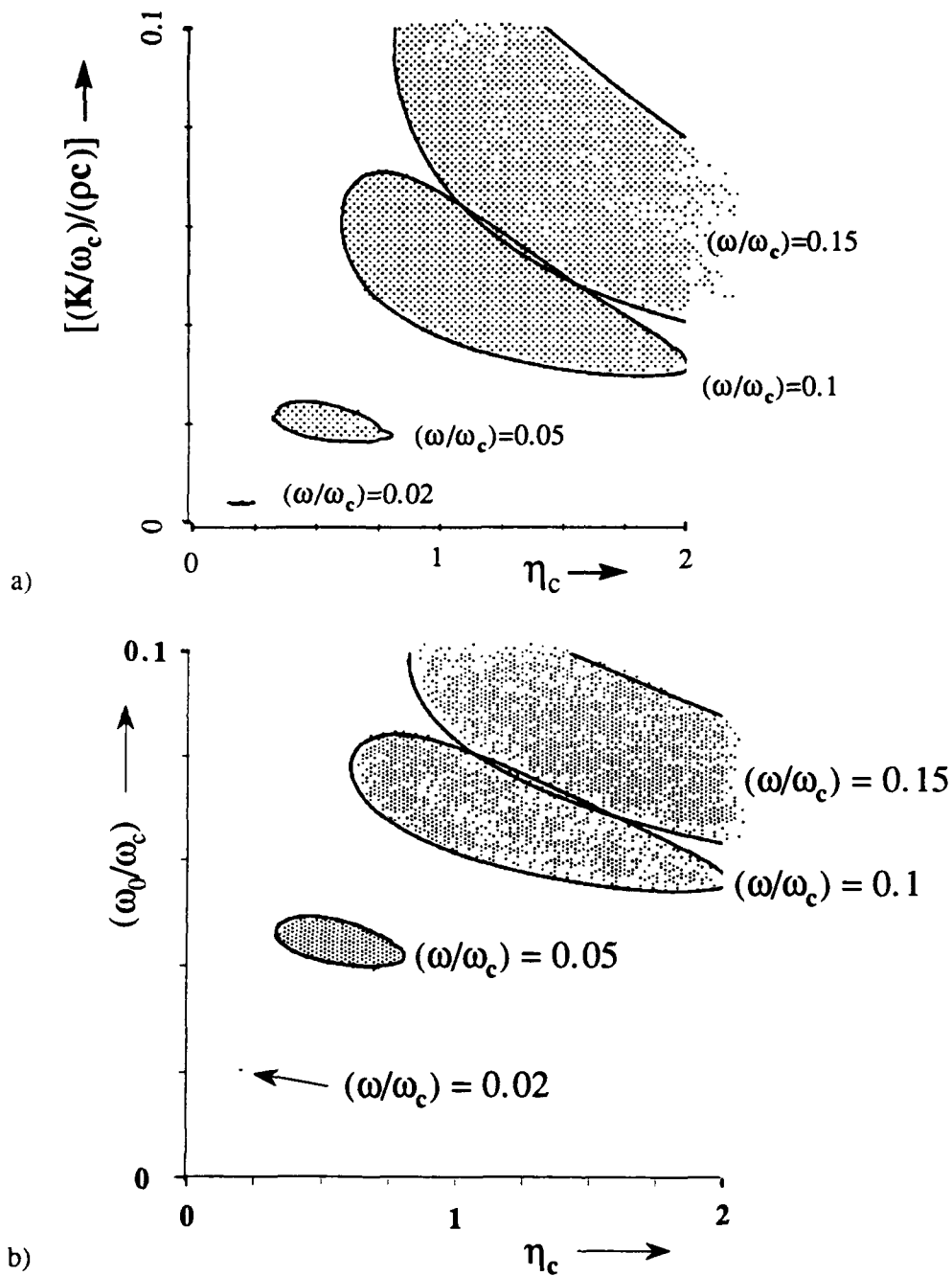   b. On the $\{(\omega_0/\omega_c), \eta_c\}$ - plane.

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Fig. 11. The regions defining the range of the material properties of the compliant layer, at fixed , values of the normalized frequency (ω/ω_c), for which the absolute value of the reflection coefficient is less than a tenth.

a. On the $\left\{ [(K/\omega_c)/(\rho c)], \eta_c \right\}$ - plane. [cf. Fig. 10a]

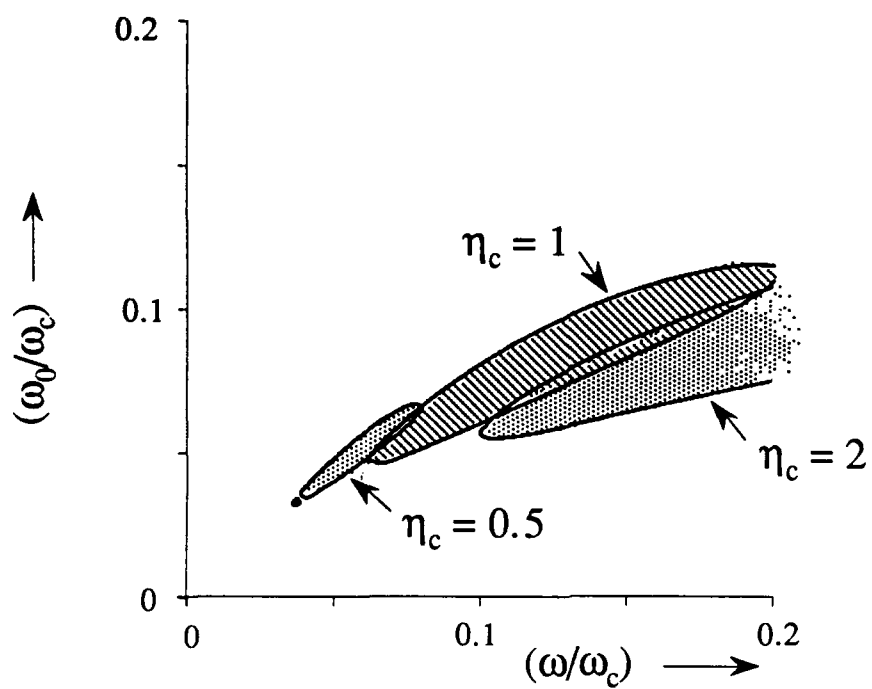b. On the $\left\{ (\omega_0/\omega_c), \eta_c \right\}$ - plane. [cf. Fig. 10b]

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Fig. 12. The regions defining the range of the normalized resonance frequency (ω_0/ω_c) and the normalized frequency (ω/ω_c), at fixed values of the loss factor η_c, for which the absolute value of the reflection coefficient is less than a tenth.

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